Quantum Mechanics Qualify Exam   Part I (50 points)   2008/10/17

1. If a particle of mass ***m*** is constrained to move in the *xy* plane on a circular orbit of radius ***R*** around the origin ***O,*** but is otherwise free, determine the energy eigenvalues and the eigenfunctions. [10 points]

2. Consider a linear harmonic oscillator with Hamiltonian

$$H = T + V = \frac{p_x^2}{2m} + \frac{1}{2}m\omega^2 x^2$$

(a) Derive the equation of motion for the expectation value $\langle x \rangle_t$ and show that it oscillates, similarly to the classical oscillator, as

$$\langle x \rangle_t = \langle x \rangle_0 \cos\omega t + \frac{\langle p_x \rangle_0}{m\omega} \sin\omega t$$ [10 points]

(b) Calculate the expectation value of x^4 for the n-th energy eigenstate of the harmonic oscillator. [8 points]

(c) Calculate the value of $\Delta x \Delta p$ for a linear harmonic oscillator in its *n*th energy eigenstate. [7 points]

3. Use the variational principle to estimate the ground-state energy of a particle in the potential **V=** ∞ for $x<0$,

V= cx for $x>0$.

Take ***x exp(-ax)*** as the trial function. [15 points]

Quantum Mechanics: Part II 2008/10/17

1. A particle of spin 1 moves in a central potential of the form

$$V(r) = V_1(r) + \frac{\vec{S}\cdot\vec{L}}{\hbar^2}V_2(r) + \frac{(\vec{S}\cdot\vec{L})^2}{\hbar^4}V_3(r)$$

   What are the values of $V(r)$ in the states $J = L+1$, L, and $L-1$? (15 points)

2. Consider the spinor $\frac{1}{\sqrt{5}}\begin{pmatrix}2\\1\end{pmatrix}$. What is the probability that a measurement of $(3S_x + 4S_y)/5$ yields the value $-\hbar/2$? (15 points)

3. An electron in the Coulomb field of a proton is in a state described by the wave function $\frac{1}{\sqrt{30}}[\psi_{100}(\vec{r}) + 2\psi_{211}(\vec{r}) - 3\psi_{210}(\vec{r}) + 4\psi_{21-1}(\vec{r})]$
   (a) What is the expectation value of the energy?
   (b) What is the expectation value of $\vec{L}^2$?
   (c) What is the expectation value of L_z? (20 points)

博士班資格考試　量子力學　**Part 3（50 points）**　**97.10.17**

Problem 1: (20 points)

(a) Write down the complete singlet and triplet wave functions for 2-electron system occupying 2 space states $\psi_a(\vec{r})\,\&\,\psi_b(\vec{r})$ and spin states u or d (write particle 1 before particle 2 to avoid confusion). (b) Write down the energy shift due to the Coulomb interaction $\dfrac{e^2}{4\pi\in_0|\vec{r}_1-\vec{r}_2|}$ in the form $\Delta E = J \pm K$ (you need write out the details of $J\,\&\,K$) and argue on physical ground that the energy is lower when the electron spins are parallel. (This is the origin of ferromagnetism.)

Problem 2: (30 points)

The partial wave amplitude can be shown to be

$f(\theta)=\dfrac{1}{2ik}\sum_{\ell=0}^{\infty}(2\ell+1)(\eta_\ell-1)P_\ell(\cos\theta)$, where $\eta_\ell=\exp(2i\delta_\ell)$ with δ_ℓ being the phase shife of the ℓ-th partial wave due to the presence of a spherical potential $V(r)$. $V(r)$ and δ_ℓ are complex if there is inelastic scattering, in which the probability is not conserved.

(a) Find the total elastic cross section $\sigma_{\ell l}$; (10 points)

(b) Given the total inelastic cross section $\sigma_{in\ell l}=\dfrac{\pi}{k^2}\sum_{\ell=0}^{\infty}(2\ell+1)\left(1-|\eta_\ell|^2\right)$,

show that $\sigma_{tot}=\sigma_{\ell l}+\sigma_{in\ell l}=\dfrac{4\pi}{k}\operatorname{Im} f(0)$; (10 points)

(c) For a totally absorbing black sphere of radius R, show that both $\sigma_{\ell l}$ & $\sigma_{in\ell l}$ are given by $\sigma_{\ell l}\approx\sigma_{in\ell l}\approx\pi R^2$, i.e. $\sigma_{tot}\approx 2\pi R^2$. (10 points)

Hint: $\eta_\ell=1$ for $\ell>kR$ ($\because$ there is no scattered partial wave when $\dfrac{\ell\hbar}{\hbar k}>R$), and

$\eta_\ell=0$ for $\ell<kR$ ($\because$ total absorption).