Cosmological constant problem #2

Theory-2002

Why is cosmological constant of the order of matter energy density?

Vacuum energy is proportional to perturbations of vacuum state

Assume that matter is dominating disturbance of vacuum, neglecting others (from gravity, curvature, quintessence, Casimir, ...)

$$E_{matter} = 3P_{matter} = \gamma T^4 \sqrt{-g}$$

liquid ^4He
$T \neq 0$

liquid ^3He
$T \neq 0$

$$E_{vacuum} = -P_{vacuum}$$

bosonic quasiparticles

$\gamma = \pi^2/30$

fermionic quasiparticles

$\gamma = 7\pi^2 N_F/120$

Equilibrium condition for isolated liquid

$$P = P_{vacuum} + P_{matter} = 0$$

$$\downarrow$$

$$E_{vacuum} = (1/3)E_{matter}$$

$$\downarrow$$

$$\frac{E_{vacuum}}{E_{theory}} = \frac{T^4}{E^4_{Planck}}$$

compare with

$$E_{vacuum} = (2\text{-}3)E_{matter}$$

In present Universe (and practically for all epochs) deviations from equilibrium vacuum are small compared to Planck scale. That is why cosmological constant is small

Cosmological Constant

Einstein equation

$$\underbrace{\frac{1}{8\pi\gamma}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right) - \Lambda g_{\mu\nu}}_{\text{gravity}} = \underbrace{T_{\mu\nu}^{\text{Matter}}}_{\substack{\text{matter} \\ \text{as source of gravity}}}$$

$$\downarrow$$

$$\underbrace{\frac{1}{8\pi\gamma}\left(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R\right)}_{\text{gravity}} = \underbrace{\Lambda g_{\mu\nu} + T_{\mu\nu}^{\text{Matter}}}_{T_{\mu\nu}^{\text{vac}} + T_{\mu\nu}^{\text{Matter}}}$$

Energy density of the vacuum

Pressure of vacuum

$$E = \Lambda \qquad P = -\Lambda$$

Equation of state for vacuum medium

$$P = -E$$

Does such cond-matter exist?

Cosmological constant problem #1

Vanishing of big cosmological constant without fine tuning

$$L = \frac{1}{16\pi G}\sqrt{-g}\,R + \sqrt{-g}\,\Lambda$$

Curvature term

Cosmological constant = vacuum energy density

$$\Lambda_{theor} = (1/2)\sum_{bosons} cp \; - \sum_{fermions} cp \;\approx\; \pm c p^4_{cut\text{-}off} \;\approx\; 10^{50 \div 120}\,\Lambda_{exp}$$

Vacuum energy of equilibrium quantum liquids

superfluid ^4He

$$\Lambda_{`theor'} = (1/2)\sum_{phonons} cp$$

$\sqrt{-g}\, E^4_{Planck}$

liquid ^4He

superfluid ^3He

$$\Lambda_{`theor'} = -\sum_{fermions} cp$$

$-\sqrt{-g}\, E^4_{Planck}$

liquid ^3He

Gibbs-Duhem relation for vacuum

$$P = -(E - \mu N) = -\Lambda_{exact}$$

vacuum pressure (P), **chemical potential** (μ), **particle number** (N), **effective cc** (Λ_{exact})

Relevant energy density of **isolated** (P=0) droplet at T=0

$$\Lambda_{exact} = E - \mu N = -P = 0$$

trans-Planckian degrees of freedom exactly compensate sub-Planckian contribution if vacuum is in equilibrium

From many-body Schrödinger equation to quantum field theory

Theory of Everything: N-particle Hamiltonian

$$H = -\frac{\hbar^2}{2m^2}\sum_{i=1}^{N}\frac{\partial^2}{\partial \vec{r}_i^{\,2}} + \sum_{i=1}^{N}\sum_{j=i+1}^{N} V(\vec{r}_i - \vec{r}_j)$$

$$\Downarrow \qquad \Psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N)$$

second quantization form

$$\hat{H} - \mu\hat{N} = \int d^3x\, \psi^+\left(-\frac{\Delta}{2m} - \mu\right)\psi + \int d^3x\, d^3y\, V(x-y)\,\psi^+(x)\,\psi^+(y)\,\psi(y)\,\psi(y)$$

quantum field theory

Numerical calculations of ground state energy

$$E = \min \frac{\int\int\int\dots\int \Psi^*(r_1,\dots,r_N)\, H\, \Psi(r_1,\dots,r_N)}{\int\dots\int \Psi^*(r_1 \dots r_N)\, \Psi(r_1,\dots,r_N)}$$

$E - \mu N = 0$ $\boxed{\text{Why?}}$

Gibbs-Duhem relation for 1 sort of particles

$$E = TS + \mu N - PV$$

energy

temperature

entropy

chemical potential

particle number

pressure

volume

Exact vacuum energy from The Theory of Everything

$$\Lambda_{exact} = E - \mu N = <\hat{H}>_{vac} - \mu<\hat{N}>_{vac}$$

The Theory of Everything

$$\hat{H} - \mu\hat{N} = \int d^3x\, \psi^+ \left(-\frac{\Delta}{2m} - \mu\right) \psi \quad +$$
$$+ \int d^3x d^3y V(x-y)\, \psi^+(x)\psi^+(y)\psi(y)\psi(x)$$

Thermodynamic Gibbs-Duhem identity for equilibrium state at T=0

$$\boxed{\Lambda_{exact} = E - \mu N = - P}$$

P is vacuum pressure

This reproduces the property of vacuum energy in QFT

$$\boxed{E_{vac} = - P_{vac}}$$

Vacuum energy
$E - \mu N$
does not depend on choice of zero !

For isolated system the pressure P is zero:

$$\boxed{\Lambda_{exact} = E - \mu N = - P} = \boxed{0}$$

If quantum system can exist in equilibrium without interaction with environment its vacuum energy is exactly zero without fine tuning

Trans-planckian degrees of freedom exactly cancel contribution from zero point energy