

Cosmological constant problem #2

Theory-2002

Why is cosmological constant of the order of matter energy density?

Vacuum energy is proportional to perturbations of vacuum state

Assume that matter is dominating disturbance of vacuum, neglecting others (from gravity, curvature, quintessence, Casimir, ...)

$$E_{\text{matter}} = 3P_{\text{matter}} = \gamma T^4 \sqrt{-g}$$

liquid ${}^4\text{He}$
 $T \neq 0$

liquid ${}^3\text{He}$
 $T \neq 0$

$$E_{\text{vacuum}} = -P_{\text{vacuum}}$$

bosonic quasiparticles

fermionic quasiparticles

$$\gamma = \pi^2/30$$

$$\gamma = 7\pi^2 N_F/120$$

Equilibrium condition for isolated liquid

$$P = P_{\text{vacuum}} + P_{\text{matter}} = 0$$

$$E_{\text{vacuum}} = (1/3)E_{\text{matter}}$$

compare with

$$\frac{E_{\text{vacuum}}}{E_{\text{theory}}} = \frac{T^4}{E_{\text{Planck}}^4}$$

$$E_{\text{vacuum}} = (2-3)E_{\text{matter}}$$

In present Universe (and practically for all epochs) deviations from equilibrium vacuum are small compared to Planck scale. That is why cosmological constant is small

Cosmological Constant

Einstein equation

$$\frac{1}{8\pi\gamma} (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) - \Lambda g_{\mu\nu} = T_{\mu\nu}^{\text{Matter}}$$

gravity

matter
as source of gravity

$$\frac{1}{8\pi\gamma} (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) = \Lambda g_{\mu\nu} + T_{\mu\nu}^{\text{Matter}}$$

gravity

$$T_{\mu\nu}^{\text{vac}} + T_{\mu\nu}^{\text{Matter}}$$

Energy
density
of the vacuum

Pressure
of
vacuum

$$E = \Lambda$$

$$P = -\Lambda$$

Equation of state for vacuum medium

$$P = -E$$

Does such cond-matter exist?

Cosmological constant problem #1

Vanishing of big cosmological constant without fine tuning

$$L = \frac{1}{16\pi G} \sqrt{-g} R + \sqrt{-g} \Lambda$$

Curvature term

Cosmological constant = vacuum energy density

$$\Lambda_{\text{theor}} = (1/2) \sum_{\text{bosons}} cp - \sum_{\text{fermions}} cp \approx \pm cp_{\text{cut-off}}^4 \approx 10^{50 \pm 120} \Lambda_{\text{exp}}$$

Vacuum energy of equilibrium quantum liquids

superfluid ^4He

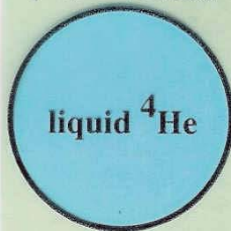
$$\Lambda_{\text{theor}}' = (1/2) \sum_{\text{phonons}} cp$$

superfluid ^3He

$$\Lambda_{\text{theor}}' = - \sum_{\text{fermions}} cp$$

$\sqrt{-g} E_{\text{Planck}}^4$

$-\sqrt{-g} E_{\text{Planck}}^4$



Gibbs-Duhem relation for vacuum

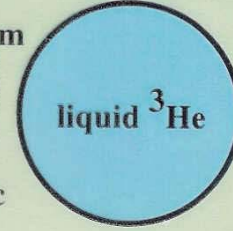
$$P = -(E - \mu N) = -\Lambda_{\text{exact}}$$

vacuum pressure

chemical potential

particle number

effective cc



Relevant energy density of isolated ($P=0$) droplet at $T=0$

$$\Lambda_{\text{exact}} = E - \mu N = -P = 0$$

trans-Planckian degrees of freedom exactly compensate sub-Planckian contribution if vacuum is in equilibrium

From many-body Schrödinger equation
to quantum field theory

Theory of Everything: N -particle Hamiltonian

$$H = -\frac{\hbar^2}{2m^2} \sum_{i=1}^N \frac{\partial^2}{\partial \vec{r}_i^2} + \sum_{i=1}^N \sum_{j=i+1}^N V(\vec{r}_i - \vec{r}_j)$$

\Downarrow $\Psi(\vec{r}_1, \vec{r}_2, \dots, \vec{r}_N)$
second quantization form

$$\hat{H} - \mu \hat{N} = \int d^3x \Psi^\dagger \left(-\frac{\Delta}{2m} - \mu \right) \Psi +$$
$$+ \int d^3x d^3y V(x-y) \Psi^\dagger(x) \Psi^\dagger(y) \Psi(y) \Psi(x)$$

quantum field theory

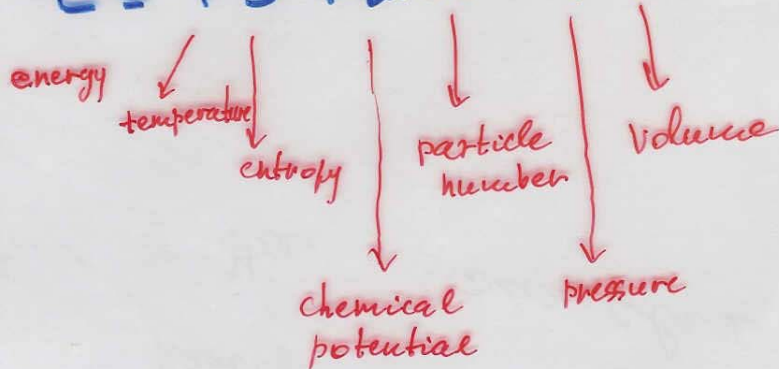
Numerical calculations of ground state energy

$$E = \min_{\Psi} \frac{\int \dots \int \Psi^*(r_1, \dots, r_N) H \Psi(r_1, \dots, r_N)}{\int \dots \int \Psi^*(r_1, \dots, r_N) \Psi(r_1, \dots, r_N)}$$

$$E - \mu N = 0 \quad \boxed{\text{Why?}}$$

Gibbs-Duhem relation
for 1 sort of particles

$$E = TS + \mu N - PV$$



Exact vacuum energy from The Theory of Everything

$$\Lambda_{\text{exact}} = E - \mu N = \langle \hat{H} \rangle_{\text{vac}} - \mu \langle \hat{N} \rangle_{\text{vac}}$$

The Theory of Everything

$$\hat{H} - \mu \hat{N} = \int d^3x \psi^\dagger \left(-\frac{\Delta}{2m} - \mu \right) \psi + \int d^3x d^3y V(x-y) \psi^\dagger(x) \psi^\dagger(y) \psi(y) \psi(x)$$

Thermodynamic Gibbs-Duhem identity for equilibrium state at $T=0$

$$\Lambda_{\text{exact}} = E - \mu N = -P$$

P is vacuum pressure

This reproduces the property of vacuum energy in QFT

$$E_{\text{vac}} = -P_{\text{vac}}$$

Vacuum energy
 $E - \mu N$
does not depend
on choice of zero !

For isolated system the pressure P is zero:

$$\Lambda_{\text{exact}} = E - \mu N = -P = 0$$

**If quantum system can exist in equilibrium
without interaction with environment
its vacuum energy is exactly zero without fine tuning**

Trans-planckian degrees of freedom
exactly cancel contribution from zero point energy