ACCELERATED PARTICLE IN QUANTUM FIELDS

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I. INTRODUCTION

- Hawking Effect 1974, 75

  Black holes emit radiation (field quanta, "particle") to infinity. temperature $T_H \sim \frac{1}{M}$ ← mass of BH.

- Unruh effect 1976

  "An accelerated detector even in flat spacetime will detect <u>field quanta</u> in the vacuum."

  "Unruh radiation"

* detector: point-like object with internal degree of freedom coupled with fields.

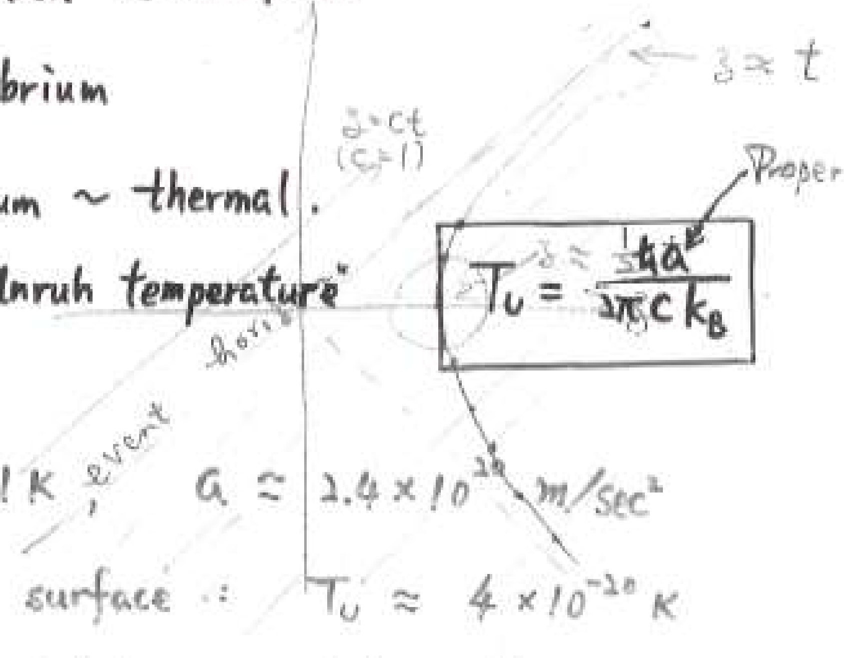
  e.g. two-level atoms, electrons, qubits.

* If acceleration is uniform

  → equilibrium

  → spectrum ~ thermal.

  with "Unruh temperature" $\boxed{T_U = \frac{\hbar a}{2\pi c k_B}}$ Proper acceleration



$T_U = 1\,K$, $a \simeq 2.4 \times 10^{20}\ m/sec^2$

Earth's surface : $T_U \approx 4 \times 10^{-20}\,K$

e^- in collider : $10^3 \sim 10^4\,K$.

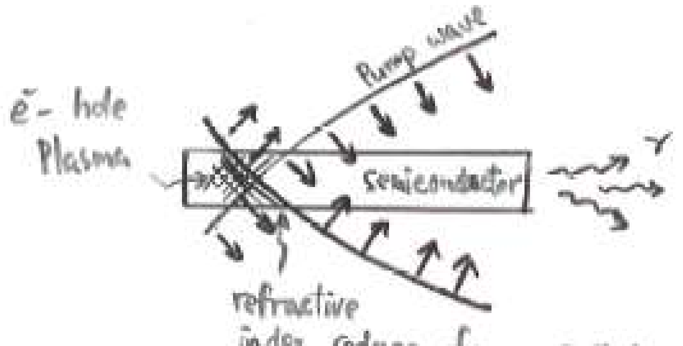
- Observing Unruh effect ?

* Bell and Leinaas 1983

de-polarization of electrons in accelerators.

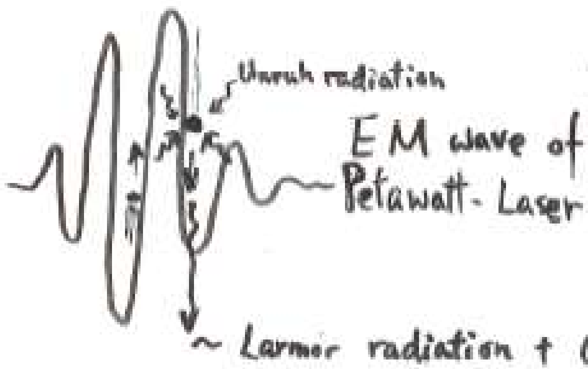
⇒ Same order of QED effect.

* Yablonovich 1988



. ~ moving mirror

* Chen (陳丕燊) and Tajima 1999



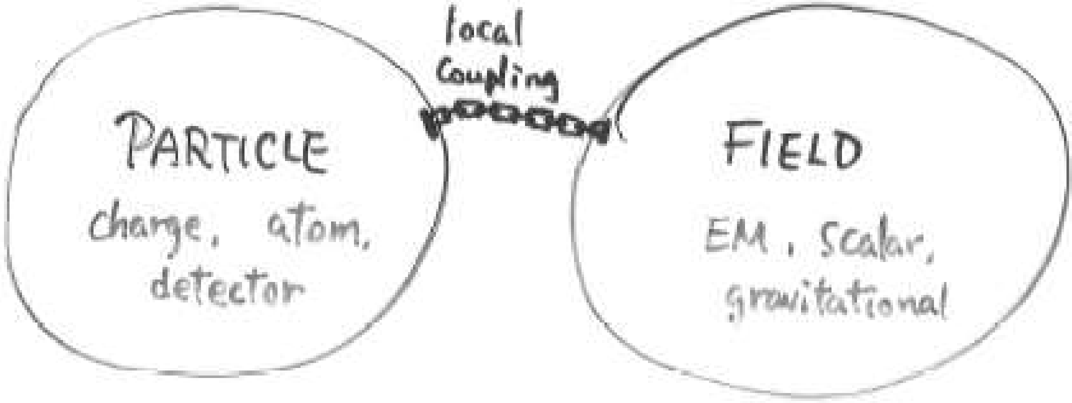
$I \sim \langle |H_2|^2 \rangle \sim \langle x^2 \rangle E^2$

J. Wang (汪治平, IAMS, AS)

Power : 10^{13} W. $a \simeq 2.9 \times 10^{24}\ m/s^2$. ⇒ $T_U \simeq 10^4 K$.

Others: Darbinyan et al), 1990 But the signal of scattered photons $\sim 10^{-3}$.

II. PARTICLE - FIELD INTERACTION



- Few degrees of freedom
- mechanics
- # of PARTICLE is conserved.

- ∞ d.o.f.
- field theory
- # of field quanta is not conserved.

Energy density of the FIELD is not enough to induce pair-creations of PARTICLE. (natural cut-off.)

Examples:

- Radiation theory in electrodynamics.
- atom-field interaction in quantum optics.
- Unruh detector theory

Methods:

- Quantum open system ($\rho_A = \mathrm{Tr}_{\text{field}}\, \rho$)
  - influence functional
  - quantum trajectory
- standard QM + QFT.

{ Accelerated detector → detect field quanta in vacuum.
{ Accelerated charge → emit radiation

<u>Q</u>: Any connection between Unruh effect and radiation theory
Any evidence of Unruh effect could be found in radiation

* Unruh and Wald, 1984,

" The inertial observer interprets the absorption of <u>Rindler particles</u> as the emission of the Minkowski particle
(field quanta recognized by the detector)

* Grove, 1986

No radiation is emitted by uniformly accelerated detector in steady state.

* Raine, Sciama and Grove, 1991

the null

(1) All retarded field can reach infinity. → No "Coulomb" field

(1+1)D harmonic oscillator + scalar field

~~Exactly~~ solvable!

(2) → No radiation. Must be balanced either by internal energy or advanced field

Follow-ups: Unruh 1992, Massar, Parentani and Brout, 1993, ...

(3) Energy flux always splits to left- and right-moving part. (right-moving never cross event horizon)

∃ vacuum polarization cloud.

HOW ABOUT IN (3+1)D ? dipole radiation.

hence Larmor formula in (1+1)D.

III. UNRUH – DEWITT "DETECTOR" THEORY IN (3+1) D

(Lin and Hu (胡比樂), 2005)

- Action (DeWitt, 1979) : $S = S_a + S_\phi + S_I$

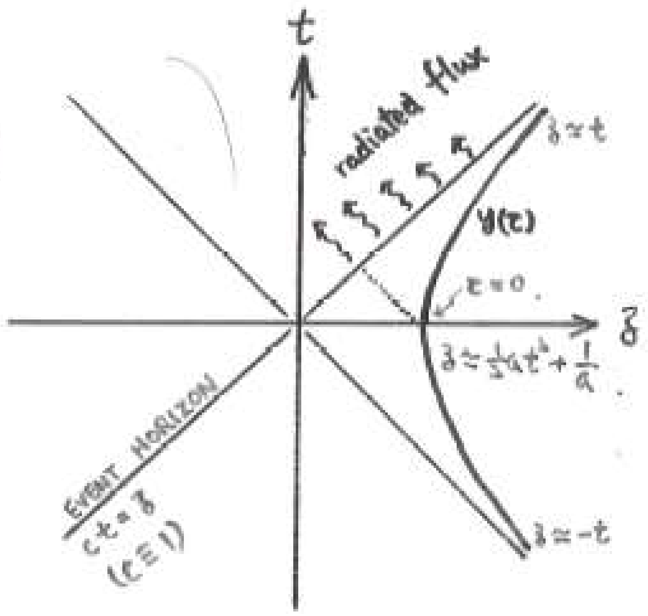
"PARTICLE" $\quad S_a = \int d\tau \frac{m_0}{2}\left[\left(\frac{dQ}{d\tau}\right)^2 - \Omega_0^2 Q^2(\tau)\right]$ Harmonic Oscillator (~ atom)

"FIELD" $\quad S_\phi = \int dt \sum_{\vec{x}} \frac{1}{2}\left[\left(\frac{\partial \phi_{\vec{x}}}{\partial t}\right)^2 - \vec{\nabla}\phi_{\vec{x}} \cdot \vec{\nabla}\phi_{\vec{x}}(t)\right]$ massless scalar field

"INTERACTION" $\quad S_I = \lambda_0 \int d\tau \int dt \sum_{\vec{x}} Q(\tau)\phi(x)\, \delta^4(x - y(\tau))$ interacting at a point.

$$\begin{array}{ccccc} & t & x & y & z \\ y(\tau) = (& \frac{1}{a}\sinh a\tau, & 0, & 0, & \frac{1}{a}\cosh a\tau) \end{array}$$

uniformly accelerated.



- In Heisenberg picture ....

<u>Initial conditions</u>

Suppose the interaction (S_I) is turned on at $\tau = t = 0$

– operators : $\hat{Q}(0) = \hat{Q}_0$, $\hat{\phi}(0, \vec{x}) = \hat{\phi}_0(t, \vec{x})$ (operators in free theory)

– quantum state : $|\tau = 0\rangle = |q\rangle \otimes |0_M\rangle$

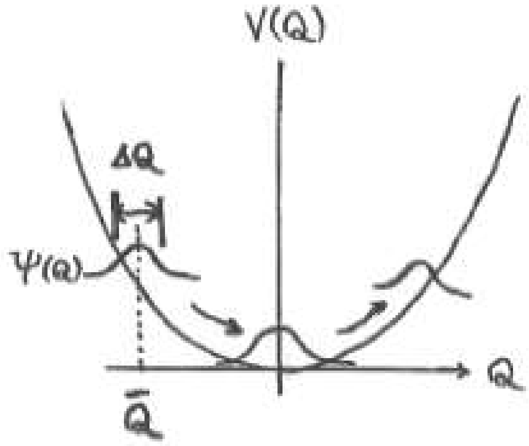
(Coherent state in S_a) (Minkowski vacuum in S_ϕ)

$\longrightarrow$ Compute $\langle \hat{Q}(\tau)^2 \rangle$, $\langle \hat{\phi}(t, \vec{x})^2 \rangle$, $\langle T_{\mu\nu}[\phi(t, \vec{x})] \rangle$, ...

RMK 1. $\psi(Q) \equiv \langle Q | \xi \rangle$

– Q-representation of coherent state $|\xi\rangle$.

$$\Delta Q = \left[\langle \hat{Q}(\tau)^2 \rangle - \bar{Q}(\tau)^2 \right]^{\frac{1}{2}}$$



RMK 2. In our case $\left(\hat{Q}(\tau) = \xi^Q(\tau)\hat{Q}_0 + \xi^P(\tau)\hat{P}_0 + \xi^\phi(\tau)\hat{\phi}(y(0)) + \xi^\pi(\tau)\hat{\pi}(y(0)) \right)$

$$\begin{aligned}
\langle \hat{Q}(\tau)^2 \rangle &= \langle 0_M | \langle \xi | \hat{Q}(\tau)^2 | \xi \rangle | 0_M \rangle \\
&= \underbrace{\langle 0_M | 0_M \rangle}_{=1} \langle \xi | \hat{Q}(\tau)^2 | \xi \rangle + \underbrace{\langle \xi | \xi \rangle}_{=1} \langle 0_M | \hat{Q}^2(\tau) | 0_M \rangle \\
&\equiv \underbrace{\langle \hat{Q}(\tau)^2 \rangle_a}_{\sim \text{atom}} + \underbrace{\langle \hat{Q}(\tau)^2 \rangle_v}_{\sim \text{vacuum}} \\
&= \bar{Q}(\tau)^2 + \underbrace{\langle \hat{Q}(\tau)^2 \rangle_a^{\text{gm}}}_{O(\hbar)} + \underbrace{\langle \hat{Q}(\tau)^2 \rangle_v}_{O(\hbar)}
\end{aligned}$$

dep. on states of $\hat{Q}$ dep. on states of $\hat{\phi}$.

RMK 3. Radiated flux through the event horizon

$$\langle T^{tt} - T^{tz}(x) \rangle \Big|_{z \to t} \sim \frac{2\lambda_0^2}{(2\pi)^2 a^4} \left\{ \frac{\langle \dot{Q}(\tau_-)^2 \rangle_{tot}}{(2t)^2 (x^2+y^2-a^{-2})^2} + \frac{(x^2+y^2)}{(x^2+y^2-a^{-2})^4} \langle [\dot{Q}(\tau_-) + aQ(\tau_-)]^2 \rangle_{tot} \right\}$$

~ monopole radiation ~ dipole radiation "Bremsstrahlung"

where $\tau_- \equiv + \frac{1}{a} \ln \left[\frac{2t}{a(x^2+y^2+a^{-2})} \right]$ field

$$\langle Q(\tau_-)^2 \rangle_{tot} = \phi_1 \langle Q(\tau_-)^2 \rangle_1 + (\textcircled{A})$$

interfering term

$\langle \phi(x)\phi(x') \rangle_{ren}$ is because $\phi(x) = \phi_0(x) + \phi_1(x)$, $\langle \phi_1(x)\phi_1(x') \rangle$ + $\langle \phi_1(x)\phi_0(x') + \phi_0(x) \rangle$, $\phi_1(x) \sim O(\lambda_0)$, ϕ_0 : free field.

$$\langle \phi(x)\phi(x') \rangle_{ren} \equiv \langle \phi(x)\phi(x') \rangle - \langle \phi_0(x)\phi_0(x') \rangle = \underbrace{\langle \phi_1(x)\phi_1(x') \rangle}_{\langle Q^2 \rangle, \langle \dot{Q}^2 \rangle} + \underbrace{\langle \phi_1(x)\phi_0(x') + \phi_0(x)\phi_1(x') \rangle}_{\textcircled{A}_{..}\ \textcircled{A}.}$$

and $\langle T_{\mu\nu}(x) \rangle \sim \lim_{x' \to x} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial x'^\nu} \langle \phi(x)\phi(x') \rangle_{ren}$

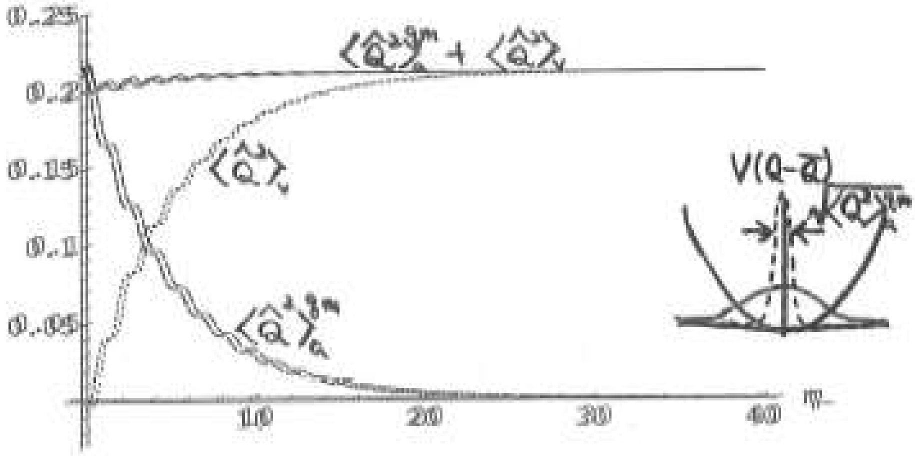


FIG. 1: The dotted line is $\langle \hat{Q}(\eta)^2 \rangle_\nu$. The dashed line is $\langle \hat{Q}(\eta)^2 \rangle_a^{qm}$, and the solid line is the sum of these two.

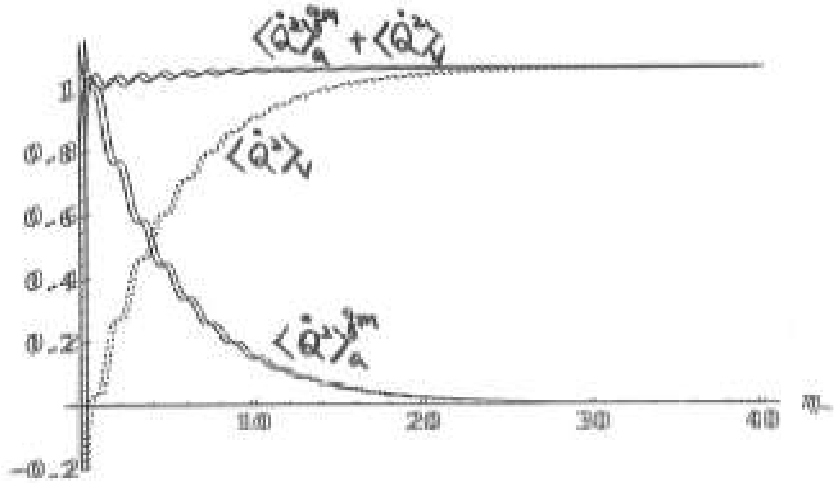
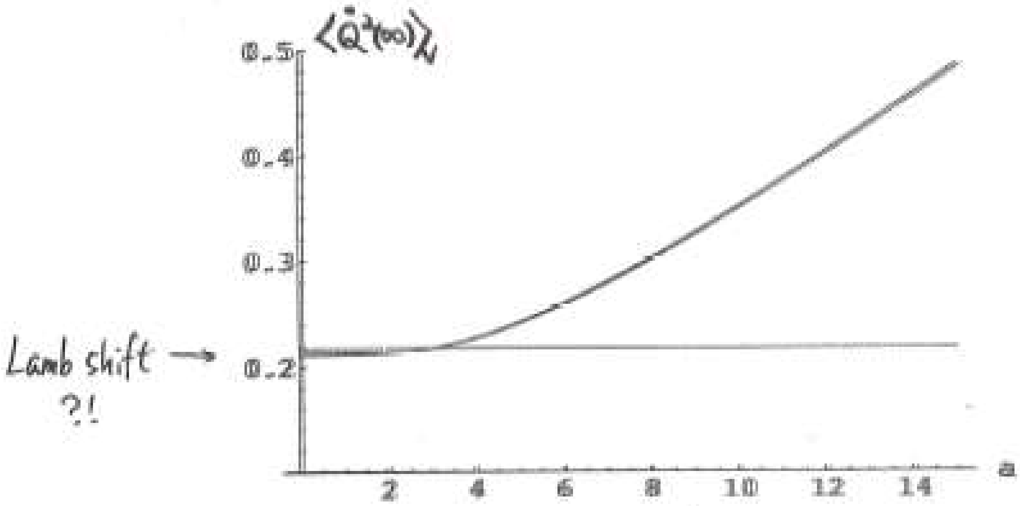
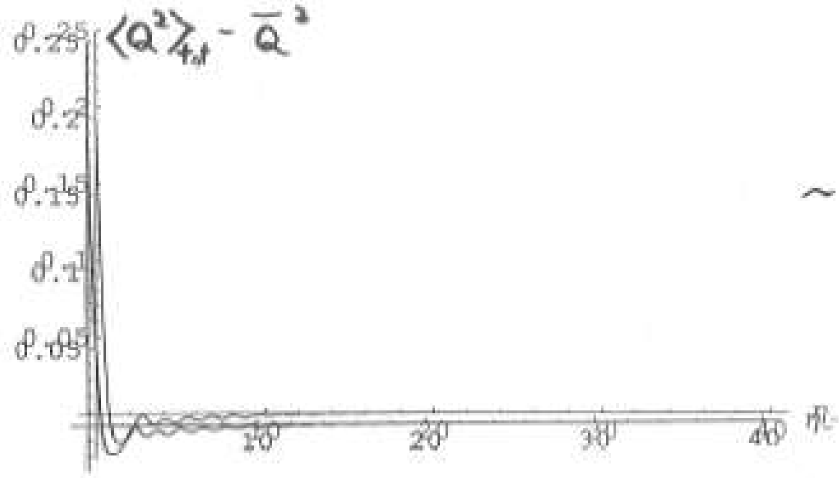


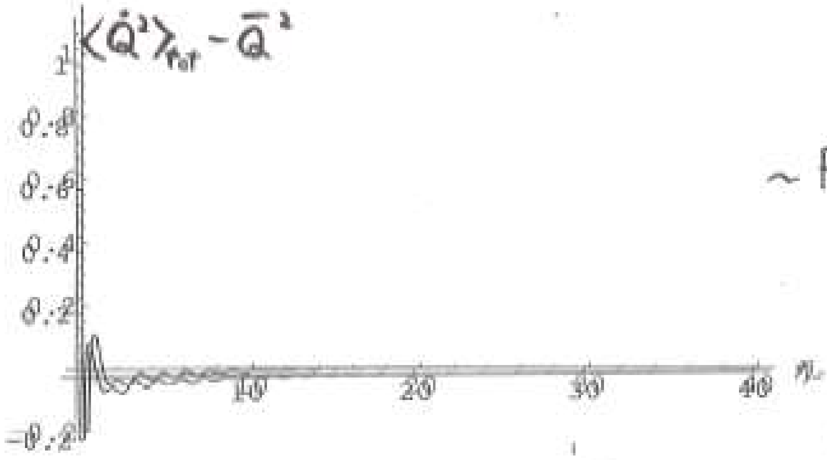
FIG. 2: Similar plot for $\langle \dot{Q}(\eta)^2 \rangle_\nu$ (dotted line), $\langle \dot{Q}(\eta)^2 \rangle_a^{qm}$ (dashed line) and their sum (solid line).





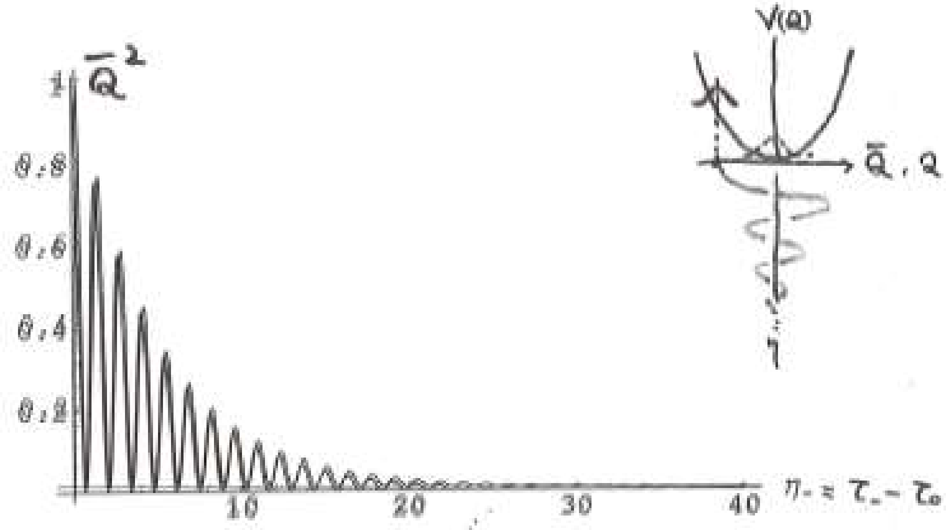
$\sim$ FIG.1 + Θ_{00}

FIG. 3: The quantum part of the total variance $[\langle Q^2 \rangle_{tot} - \bar{Q}^2]$ near the event horizon for the detector.



$\sim$ FIG.2 + Θ_{11}

FIG. 4: The quantum part of the total variance $[\langle \dot{Q}^2 \rangle_{tot} - \dot{\bar{Q}}^2]$ with the same parameters.



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Summary

1. Choosing vacuum ~ choosing boundary conditions.

2. One observer's vacuum may not be another's no-particle (field quanta) state.

   "Particle" number is frame-dependent.

3. Accelerated atoms in Minkowski vacuum "see particles". (Overall effect seems to be a shift of ground state only.)

4. However, the evidence of the Unruh effect in radiation emitted by the accelerated atom is totally suppressed in steady state.. Hence Unruh temperature cannot be observed by inertial observers.

5. "Quantum radiation" can only be seen in transient.

6. For electron - EM field system : to be confirmed.