String Theory and its high-energy limit

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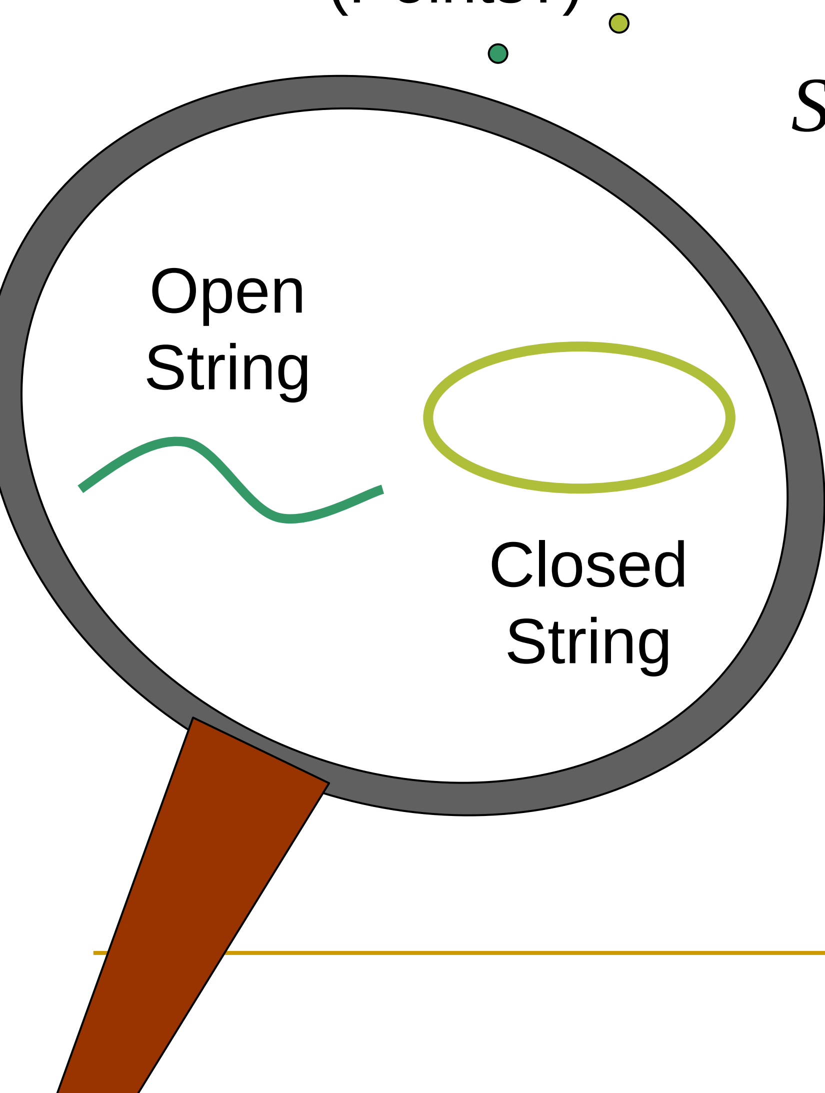
Contents of Today's Talk

1. Brief review of String Theory
2. Spectrum of String Theory
3. High-energy limit of String Theory
4. Linear relations among H.E. amplitudes
5. Surviving particle
6. Summary and Future problem

Based on works in collaboration with
C.T. Chan, J.C. Lee, P.M. Ho and Y. Yang

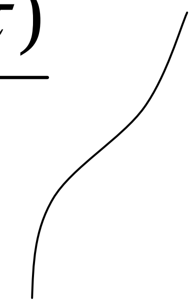
Basic idea of String Theory

Elementary particles
(Points?)



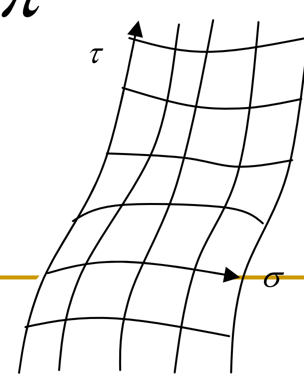
Length of World Line

$$S = -\frac{1}{2}\int d\tau \sqrt{\frac{dX^{\mu}(\tau)}{d\tau}\frac{dX_{\mu}(\tau)}{d\tau}}$$



Area of World Sheet

$$S = -\frac{1}{\pi}\int d\tau d\sigma \sqrt{\det \partial_a X^{\mu} \partial_b X_{\mu}}$$



$a, b = 1,2$

$$\partial_1 \equiv \frac{\partial}{\partial \sigma}$$

$$\partial_2 \equiv \frac{\partial}{\partial \tau}$$

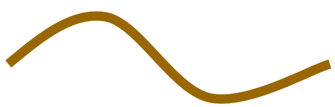
Spectrum of String Theory

- an oscillation of string = a particle

  ⇒ Infinitely many kinds of particles

  (gauge bosons, graviton…)

- Oscillation of string is origin of mass and spin.



Lower mass, Lower spin

Higher mass, Higher spin

- Massive states are too massive! Planck mass $\sim 10^{19}\,\mathrm{GeV}$
  - Only "massless" states are accessible by accelerator.
  - However higher mass and spin states are required from consistency.

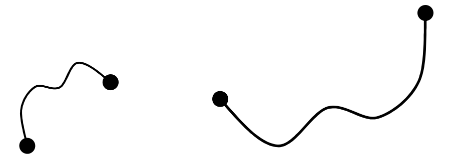
Properties of String Theory

- Quantum theory with graviton and gauge boson.
- Huge gauge symmetry.
- Unique Interaction and no free parameter.
- Higher space-time dimensions, 10D or 26D.
- Only perturbatively defined.
- Perturbatively, infinitely many vacua.

⇨ Unfortunately, string theory "predicts" infinitely many low-energy theories.

⇨ We must understand string theory beyond perturbation!

Bosonic Open String Theory



- Simplest string theory.
- String has end points.
- No fermion. (not for describing our world)
- Space-time is 26 dimensional.
- Infinitely many particles including tachyon (scalar), photon and higher spin fields.
- Described by 2-dimensional world sheet theory

$$S = -\frac{1}{\pi}\int d\tau d\sigma \sqrt{\det \partial_a X^\mu \partial_b X_\mu}$$

World Sheet theory (2D field theory)

$$S = -\frac{1}{\pi}\int d\tau d\sigma \sqrt{\det \partial_a X^\mu \partial_b X_\mu}$$

γ_{ab} :2D metric

Classically equivalent

$$S = -\frac{1}{2\pi}\int d\tau d\sigma (-\gamma^{1/2})\gamma^{ab}\partial_a X^\mu \partial_b X_\mu$$

General coordinate tr. inv. $\frac{\partial \sigma'^c \partial \sigma'^d}{\partial \sigma^a \partial \sigma^b}\gamma'_{cd} = \gamma_{ab}$ $\quad \sigma_1 \equiv \sigma, \quad \sigma_2 \equiv \tau$

Local scale tr. Inv. $\gamma'_{ab}(\tau,\sigma) = \exp(2\omega(\tau,\sigma))\gamma_{ab}(\tau,\sigma)$

2D gravity with local scale gauge inv.

We must fix these gauges!

Gauge fixing

- We take conformal gauge: $\gamma_{ab}(\tau,\sigma)=\delta_{ab}$
  (γ_{ab} was gauged away.)
- Action become a 2D free scalar field theory.

$$S=-\frac{1}{2\pi}\int d\tau d\sigma \partial_a X^{\mu}\partial^a X_{\mu}$$

- However, we missed eq. of motion for γ_{ab} :

$$T_{ab}(\tau,\sigma)=0$$

T_{ab} :2D energy momentum tensor

$$T_{ab}=-2\partial_a X^{\mu}\partial_b X_{\mu}+\delta_{ab}\partial^c X^{\mu}\partial_c X_{\mu}$$

Mode expansion

- Introduce complex coordinate $z = -\exp(-i\sigma + \tau)$
- Mode expansion of X^μ (Fourier expansion)

$$X^\mu(z) = x^\mu - \frac{i}{2} p^\mu \log|z|^2 + \frac{i}{2} \sum_{m \neq 0} \frac{\alpha_m^\mu}{m} (z^{-m} + \bar{z}^{-m})$$

$$[x^\mu, p^\nu] = i\eta^{\mu\nu} \qquad [\alpha_m^\mu, \alpha_n^\nu] = m\delta_{m+n}\eta^{\mu\nu}$$

- Mode expansion of energy momentum tensor

$$T(z) = \sum_{m=-\infty}^{\infty} \frac{L_m}{z^{m+2}}$$

L_m :Virasoro operator

$$[L_m, L_n] = (m-n)L_{m+n} + \frac{D}{12}(m^3 - m)\delta_{m+n}$$:Virasoro algebra

Physical condition (Gupta-Bleuler)

Operator equation (missing eq. of motion)

$$\langle \phi' | T(z) | \phi \rangle = 0$$

Physical condition for 1-string state

$$\begin{cases} L_n | \phi \rangle = 0 \quad \text{for} \quad n > 0 \\ (L_0 - 1) | \phi \rangle = 0 \end{cases}$$

$$L_n = \frac{1}{2} \sum_{m=-\infty}^{\infty} \alpha^{\mu}_{n-m} \alpha_{\mu m}$$

$$L_0 = \frac{1}{2} p^2 + \sum_{m=1}^{\infty} \alpha^{\mu}_{-m} \alpha_{\mu m}$$

On shell condition

$$(p^2 + M^2) | \phi \rangle = 0$$

$$M^2 = 2 \left(\sum_{m=1}^{\infty} \alpha^{\mu}_{-m} \alpha_{\mu m} - 1 \right)$$

Physical Spectrum (Examples)

- Tachyon $\left| p \right\rangle$ $\quad M^2 = -2$
- Massless gauge boson

$$\varepsilon_\mu \alpha_{-1}^\mu \left| p \right\rangle \qquad M^2 = 0, \quad \varepsilon_\mu p^\mu = 0$$

- 1st massive mode

$$(\varepsilon_{\mu\nu} \alpha_{-1}^\mu \alpha_{-1}^\nu + \varepsilon_\mu \alpha_{-2}^\nu) \left| p \right\rangle \qquad M^2 = 2$$

$$\varepsilon_\mu = -\varepsilon_{\mu\nu} p^\nu, \qquad \eta^{\mu\nu} \varepsilon_{\mu\nu} = 2\varepsilon_{\mu\nu} p^\mu p^\nu$$

- 2nd massive mode…

As mass level becomes large, number of physical particles at same mass level grows up exponentially.

Zero-norm States (Examples)

- Among physical states, there are special states which has zero-norm.
  - Massless $p_\mu \alpha_{-1}^{\mu} \left| p \right\rangle$
  - 1st massive $\varepsilon_\mu (p_\nu \alpha_{-1}^{\mu} \alpha_{-1}^{\nu} + \alpha_{-2}^{\mu}) \left| p \right\rangle$

    $[(\eta_{\mu\nu} + 3 p_\mu p_\nu) \alpha_{-1}^{\mu} \alpha_{-1}^{\nu} + 5 p_\mu \alpha_{-2}^{\mu}] \left| p \right\rangle$
  - 2nd massive… $[2\varepsilon_{\mu\nu} \alpha_{-1}^{\mu} \alpha_{-2}^{\nu} + p_\lambda \varepsilon_{\mu\nu} \alpha_{-1}^{\lambda} \alpha_{-1}^{\mu} \alpha_{-1}^{\nu}] \left| p \right\rangle \cdots$
- These zero norm states are responsible to Space-Time gauge symmetry.

$$\varepsilon_\mu \alpha_{-1}^{\mu} \left| p \right\rangle \sim \varepsilon_\mu \alpha_{-1}^{\mu} \left| p \right\rangle + p_\mu \alpha_{-1}^{\mu} \left| p \right\rangle$$

$$A_\mu(x) \sim A_\mu(x) + \partial_\mu \lambda(x)$$

Zero-norm States

- Spurious state $\begin{cases} \left| spu \right\rangle = L_{-n} \left| \chi \right\rangle \quad \text{for } \exists n > 0 \\ (L_0 - 1) \left| spu \right\rangle = 0 \end{cases}$
- By definition, spurious state is orthogonal to all physical states.

$$\left\langle spu \middle| phys \right\rangle = \left\langle \chi \middle| L_{-n}^{\dagger} \middle| phys \right\rangle = \left\langle \chi \middle| L_{n} \middle| phys \right\rangle = 0$$

- "Spurious and physical" state is orthogonal to all physical states including itself.

→ Zero-norm state

Calculation of String Amplitudes

- Every state corresponds to vertex operator: $V(p)$
  - Tachyon $V_{\text{tachyon}}(p) = e^{ip\cdot X}$
  - Massless $V_{\text{massless}}(p) = \varepsilon_{\mu}\partial X^{\mu} e^{ip\cdot X}$
  - 1st massive $V_{\text{1st}}(z) = (\varepsilon_{\mu\nu}\partial X^{\mu}\partial X^{\nu} + \varepsilon_{\mu}\partial^{2} X^{\mu})e^{ik\cdot X}$
- Based on Wick contraction of vertex operator, we can calculate string amplitudes.

$$A = \langle V_1(p_1)V_2(p_2)V_3(p_3)V_4(p_4)\rangle$$

- Summation of Feynman diagram is replaced by summation of 2D Riemann surface.
- Interaction is consistent with gauge structure.

String Field Theory

- For bosonic open string, it is known 2nd quantized description of string.

$$S = -\frac{1}{2}\Phi \cdot Q_B \Phi - \frac{g}{3}\Phi \cdot \Phi * \Phi$$

- String Field Theory is a field theory with infinitely many fields and non-local interaction.

$$S = \frac{1}{2}\int dx^{26}\left(T(x)([]+2)T(x) + A^{\mu}[]A_{\mu} + \cdots\right)$$

$$-\frac{g}{3}\int \prod_{i=1}^{3} \frac{dk_i^{26}}{(2\pi)^{26}}(2\pi)^{26}\delta(k_1+k_2+k_3)\left(\frac{3^4\sqrt{3}}{64}\right)^{1-\frac{1}{6}\sum k_i^2} T(k_1)T(k_2)T(k_3)+\cdots$$

Motivation of High-energy limit

It is believed that,

massive mode of string acquired their mass via some spontaneous symmetry breaking.

If so, what is the hidden symmetry?

The information of the hidden symmetry might be restored in the high-energy region.

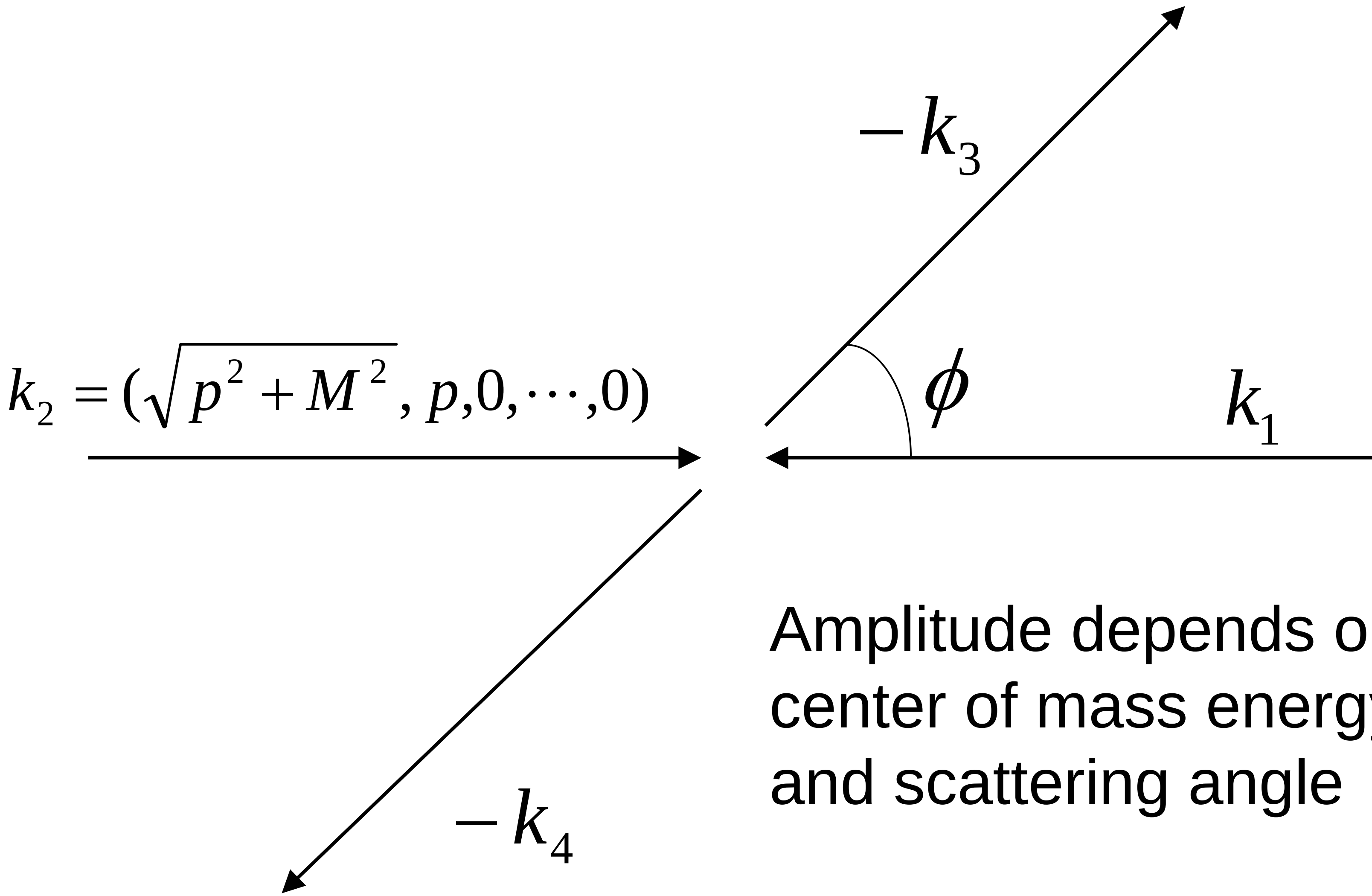
And, hopefully, the information helps us to define string theory without regard to perturbation.

By Gross et.al. 89'

How to take a high-energy limit?

- It is not clear how we should take a high-energy limit.
- High-energy limit of world sheet theory?
- High-energy limit of space-time theory?
  - Higher spin gauge theory?
- Here, we simply consider high-energy limit of 4-point amplitudes of strings with fixed scattering angle.

Kinematics



Amplitude depends on center of mass energy E and scattering angle ϕ

We must specify polarizations of particles.

Our definition of leading order.

- We focus on leading amplitudes at same mass level.
- We fix 3-particles and replace the other particle by particles which has same mass.
- For example, we consider

$$A = \left\langle V_{\text{tachyon}} V_2 V_{\text{tachyon}} V_{\text{tachyon}} \right\rangle$$

V_2 :Various particles at same mass level

- We call the amplitude leading if it is leading among same mass particles.

Polarizations

- To represent polarizations, we use this basis.

$$e^P = (\sqrt{p^2 + M^2}, p, 0, \ldots, 0)/M \qquad e^P \cdot e^P = -1$$

$$e^L = (p, \sqrt{p^2 + M^2}, 0, \ldots, 0)/M \qquad e^L \cdot e^L = 1$$

$$e^{T_i} = (0, 0, \ldots, 1, \ldots, 0) \qquad e^{T_i} \cdot e^{T_i} = 1$$

- For operators, $\alpha_{-n}^A \equiv e^A \cdot \alpha_{-n} \quad (A = P, L \ \ or \ \ T_i)$

$$\alpha_{-1}^{T_i} | p \rangle \quad \alpha_{-1}^P \alpha_{-1}^{T_i} | p \rangle \quad \alpha_{-1}^L \alpha_{-2}^L | p \rangle \quad \alpha_{-4}^{T_i} | p \rangle \quad \alpha_{-1}^L \alpha_{-1}^L \alpha_{-2}^L \alpha_{-4}^P \alpha_{-4}^{T_i} | p \rangle$$

Naïve observation of high-energy limit

$$A \sim \left\langle V_{\text{tachyon}}(p_1) V_2(p_2) V_{\text{tachyon}}(p_3) V_{\text{tachyon}}(p_4) \right\rangle$$

$$V_{\text{tachyon}}(p_i) = e^{ip_i \cdot X} \qquad V_2(p_2) = V_{\text{massless}}(p_2) = e^A \cdot \partial X e^{ip_2 \cdot X}$$

Wick contraction

$$(e^A \cdot \partial X e^{ip_2 \cdot X}) e^{ip_i \cdot X} \sim e^A \cdot p_i \, (e^{ip_2 \cdot X} e^{ip_i \cdot X})$$

Direction parallel to scattering plane

$$e^P \cdot p_i \sim E^2, \quad e^L \cdot p_i \sim E^2$$

$$e^T \cdot p_i \sim E, \quad e^{T_i} \cdot p_i \sim 0$$

Naively, $\alpha_{-1}^L \cdots \alpha_{-1}^L \left| p_2 \right\rangle$ is leading.

$$(\alpha_{-1}^P \cdots \alpha_{-1}^P \left| p_2 \right\rangle)$$

Energy suppression

- However, previous observation is not true.
- There is cancellation of (apparently) leading amplitude with α_{-1}^{P} or α_{-1}^{L}.
- In the rest of this talk,
  - Identify particles which give leading amplitudes
  - Show linear relations among leading amplitudes

by using decoupling of high-energy zero-norm state (or spurious state.)

$$(\alpha_{-1}^{T})^{n-2m-2q}(\alpha_{-1}^{L})^{2m}(\alpha_{-2}^{L})^{q}\left|p\right\rangle$$

$$A^{(n,2m,q)}=\left(\frac{-1}{M}\right)^{2m+q}\left(\frac{1}{2}\right)^{m+q}(2m-1)!!\,A^{(n,0,0)}$$

Stringy Word Identity

C.T.Chan and J.C.Lee

C.T.Chan, J.C.Lee and P.M.Ho

- Decoupling of zero-norm states leads to linear relations of amplitudes.

$$A = \left\langle V_1(p_1) V_2^{\mathrm{ZNS}}(p_2) V_3(p_3) V_4(p_4) \right\rangle = 0$$

$$\Rightarrow \quad \sqrt{2} A^{PL} + A^{L} = 0, \qquad \frac{5}{2} A^{PP} + \frac{1}{2} A^{LL} + \frac{1}{2} A^{TT} + \frac{5}{\sqrt{2}} A^{P} = 0$$

$$\left| zns \right\rangle = (\sqrt{2} \alpha_{-1}^{P} \alpha_{-1}^{L} + \alpha_{-2}^{L}) \left| p \right\rangle$$

- In the high-energy limit, $e^{P} \sim e^{L} \sim (p, p, 0, \cdots, 0)$ therefore replace P by L.

$$\sqrt{2} A^{LL} + A^{L} = 0, \quad 3A^{LL} + \frac{1}{2} A^{TT} + \frac{5}{\sqrt{2}} A^{L} = 0$$

- Linear relation $A^{TT} : A^{LL} : A^{L} = 4 : 1 : -\sqrt{2}$

Our progress

C.T.Chan, J.C.Lee, P.M.Ho, S.T and Y.Yang

- They derived linear relations up to 3rd massive level.

We derived the relations for every mass level!

- It was not really justified to replace $P \to L$ where $1/E$ contribution was simply ignored.

We justified it on a simple *Assumption*!

Our result

- Only states of the form $(\alpha_{-1}^{T})^{n-2m-2q}(\alpha_{-1}^{L})^{2m}(\alpha_{-2}^{L})^{q}\left|p\right\rangle$ are leading among particles with $M^{2}=2(n-1)$.
- All leading amplitudes are linearly related:

$$V^{(n,2m,q)}\equiv(\partial X^{T})^{n-2m-2q}(\partial X^{L})^{2m}(\partial^{2}X^{L})^{q}e^{ip\cdot X}$$

$$A^{(n,2m,q)}\equiv\left\langle V_{1}V_{2}^{(n,2m,q)}V_{3}V_{4}\right\rangle$$

Linear relations

$$A^{(n,2m,q)}=\left(\frac{-1}{M}\right)^{2m+q}\left(\frac{1}{2}\right)^{m+q}(2m-1)!!\,A^{(n,0,0)}$$

Comments

- We also derived same linear relation from other methods:
  - "Dual" calculation by Virasoro constraints
  - "Ad hoc" calculation by simplified Virasoro op
  - "sample" calculation by saddle point method
- For multi-tensor amplitudes, if we check the *Assumption*, we can conclude same results.

$$A = \left\langle V_{\text{tachyon}} V_2 V_{\text{tachyon}} V_{\text{tachyon}} \right\rangle$$

$$\Downarrow$$

$$A = \left\langle V_{\text{tensor}} V_2 V_{\text{tensor}} V_{\text{tensor}} \right\rangle$$

Summary

- We obtained infinitely many linear relations at same mass level by using decoupling of high-energy zero-norm state (or spurious state.)

Linear relations

$$A^{(n,2m,q)} = \left(\frac{-1}{M}\right)^{2m+q} \left(\frac{1}{2}\right)^{m+q} (2m-1)!!\, A^{(n,0,0)}$$

- We identified one surviving particle in the high-energy limit at given mass level.
  - Originally, number of particles grows exponentially. However, our result indicates, in high-energy limit, the growth of particles is much more moderate.

Future problems

- Generalization
  - n-point, multi-tensor amplitude
  - closed or super string theories
- Hidden symmetry?
  - “High-energy” effective theory?
  - Symmetry generator?
  - 2D string
  - String field theory?
- High-temperature limit?

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