Chapter 2: Gaussian Optics - paraxial optics
Mainly we will discuss
- Analysis based on paraxial optics
- Simple ray tracing
- Aperture and stop
- homeworks
Analysis based on paraxial optics

- Paraxial approximation
- Cardinal points
- Paraxial ray tracing
  - YNU ray tracing
  - YUI ray tracing
- Matrix Optics
- Paraxial constants
- Image equations
- Specification of conjugate distances
- Lens setup
Apertures and Pupils

- Radiometric concepts and terminology
  - Radiance conservation theorem
  - Irradiance by a flat circular source
  - Cos4 law
  - Vigentting
  - Computation of irradiance

- Stops and Pupils
  - Specifying aperture and field of view

- Optical System Layout
  - Thin lens
  - Photographic objective
  - Magnifier
  - Telescope
  - Relay system
  - Telecentric lens

- Specifying lens apertures
  - Special apertures
• Centered optical system
  – Optical axis
  – Basic law
  \[ \sin \theta = \sin \theta' \Rightarrow n \theta = n' \theta' \]
  – Plane of incidence
    • Formed by incident and refracted rays
  – Merdinal ray (子午線)
    • On the plane of incidence
    • with optical axis
  – Skew ray (歪斜線)
    • Optical axis is not on the plane of incidence
Approximation: a quick way for ray tracing

- Now let us take a non-flat surface

\[ n'e' = n'e + G \]

\[ G = n'(p \cdot e) - n(p \cdot e) \]

\[ = n'\cos I' - n \cos I \]
Cause of aberration

- Aberration: the difference between real ray and paraxial ray
Cardinal point (基點)

• Using cardinal points to characterize the performance of paraxial optics
  – Object space
  – Image space

• Two points are enough to determine the position of image

- Focal points
- Principal points
- Nodal points
Paraxial optical system and aplanatic optical system

P, P': principal points (O and O' off-axis where P and P' on axis)
Paraxial optical system and aplanatic optical system

OSLO used aplanatic Ray aiming

principal planes changes to principal surface
Paraxial optical system and aplanatic optical system

- For an aplanatic lens, the effective refracting surface for ray coming from infinity is a sphere centered on the second focal point
  - Most real lens more closely resemble aplanatic system than paraxial systems
- Note: an aplanatic lens obeys the Abbe sine condition. This condition was put into the calculation in common ray-tracing and thus to achieve an aplanatic ray aiming (instead of paraxial ray aiming).
Nodal points: without angular deviation
Unit positive angular magnification
Once the locations of the cardinal points of a system are known, it is straightforward to find how the system transforms rays from object to image space.
Paraxial ray tracing

\[
u = \frac{y - y_{-}}{t} (y = y_{-} + tu)\]

\[I = U + \theta; I' = U' + \theta\]

\[i = \frac{y}{r} + u = yc + u\]

\[i' = \frac{y}{r} + u' = yc + u'\]

\[n_i = n'i'\]

\[u' = \frac{nu - y\phi}{n'}\]

power \( \phi' = c(n' - n) \)
### YNU ray tracing

The idea of "SOLVE" is illustrated here with equations and tables.

#### Equations

- \( n'u' = nu - y\phi \)
- \( y = y_0 + \frac{t}{n} (nu) \)

#### Tables

**Table 1:**

<table>
<thead>
<tr>
<th>c</th>
<th>t</th>
<th>n</th>
<th>-( \phi )</th>
<th>( u/n )</th>
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#### Calculations

- \( c = \frac{n'u' - nu}{y(n - n')} \) (Angle solve)
- \( t = \frac{y - y_0}{u} \) (Height solve)
• Usually in ray tracing, we need to check two rays
  – (1) so called “a ray”: axial rays (marginal paraxial ray)
  – (2) so called “b ray”: principal ray (chief ray)
  – In last viewgraph, the subscripts of y (a, b) mean a ray and b ray correspondingly.
YUI ray tracing

\[
y = y_0 + tu
\]
\[
i = u + yc
\]
\[
u' = u + \left(\frac{n}{n'} - 1\right)i
\]

In OSLO use “pxt all” to display this data.

<table>
<thead>
<tr>
<th>SRF</th>
<th>PY</th>
<th>PU</th>
<th>PI</th>
<th>PYC</th>
<th>PUC</th>
<th>PIC</th>
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<td>1.00132</td>
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homework

- Using visual C++ or the other computer language to implement the ynu ray tracing and yui ray tracing
- And then, calculate a case of achromat lens
- Using OSLO LT, and “pxt all” command to compare your result

<table>
<thead>
<tr>
<th>Material</th>
<th>&lt;radius value&gt;</th>
<th>&lt;thickness value&gt;</th>
<th>&lt;diameter&gt;</th>
<th>Note: unit in inch</th>
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<tr>
<td>SF-8</td>
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<tr>
<td>AIR</td>
<td>-37.6553</td>
<td>19.631</td>
<td>3.41</td>
<td></td>
</tr>
<tr>
<td>Cemented achromat</td>
<td>F-number: f/6</td>
<td>Focal length: 20 in</td>
<td>Note: unit in inch</td>
<td></td>
</tr>
</tbody>
</table>
Matrix Optics

Translation matrix

\[
\begin{bmatrix}
y_j \\
n_j u_j
\end{bmatrix} =
\begin{bmatrix}
1 & t_j / n_j \\
0 & 1
\end{bmatrix}
\begin{bmatrix}
y_{j-1} \\
n_j u_j
\end{bmatrix}
\]

Refraction matrix

\[
\begin{bmatrix}
y_j \\
n_j' u_j'
\end{bmatrix} =
\begin{bmatrix}
1 & 0 \\
-\phi & 1
\end{bmatrix}
\begin{bmatrix}
y_j \\
n_j u_j
\end{bmatrix}
\]
Transfer Matrix

\[
\begin{bmatrix}
  y_j \\
  n'_j u'_j \\
\end{bmatrix}
= 
\begin{bmatrix}
  A & B \\
  C & D \\
\end{bmatrix}
\begin{bmatrix}
  y_1 \\
  n_1 u_1 \\
\end{bmatrix}
\]

Determinant L

\[
L_{ab} = n_1 \left( y_{b1} u_{a1} - y_{a1} u_{b1} \right)
\]

Largange invariant

(again, here subscripts a and b correspond a ray and b ray)
But you do not have to use a ray and b ray to have a Lagrange invariant
Lagrange’s Law

• How to calculate the magnification?
  – $m = h'/h$
  – Ray tracing (not good)

• Using Lagrange invariant
  – $L_{ab} = n_1 y_{bo} u_{a1} = h n_1 u_{a1}$ (axial ray, zero height in object space)
  – $L_{ab} = n_k y_{bk} u_{ak} = h' n_k u_{ak}$ (axial ray, zero height on the image plane)
  – Lagrange invariant $h n_1 u_{a1} = h' n_k u_{ak}$
  – $M = h'/h = n_1 u_{a1} / n_k u_{ak}$
Paraxial Constants

- OSLO computes seven numbers (paraxial constants)
  - Effective focal length (EFL)
  - Lagrange (paraxial) invariant (PIV)
  - Lateral magnification (TMAG)
  - Gaussian image height (GIH)
  - Numerical aperture (NA)
  - F-number (FNB)
  - Petzval radius (PTZRAD)
  
  \[
  f' = -\frac{eh}{u^e + u^a h}
  \]
  
  \[
  PIV = L_{ab} = hnu^a
  \]
  
  \[
  TMAG = m = \frac{nu^a}{n' u^a}
  \]
  
  \[
  GIH = h' = \frac{nu^a}{n' u^a} h
  \]
  
  \[
  NA = \frac{NAO}{TMAG}
  \]
  
  \[
  NAO = n \sin U = \frac{nu^a}{\sqrt{1 + u^2_a}}
  \]
  
  - The radius of curvature of the surface on which an image of an extended plan object would be formed (not a paraxial quantity, actually)
So, in designing a code for optical system design, at least

- You need to know how to implement ynu and yui ray tracing \textit{(this is simple)}
- You need to know how to put Abbe sine conditions to have an aplanatic ray aiming \textit{(this is simple)}
- You need to know how to use Lagrange invariant to calculate fast \textit{(this is simple)}
- At least, seven paraxial constants should be implemented \textit{(this is simple)}
- You need to know how to implement “Solve” and “Pickup” to meet some design requirement and optimization needs \textit{(this is somehow difficult; you need to consult “numerical recipe”. But, it can be done.)}
Specification of conjugate distance

In OSLO, you can specify object and image distance referred to cardinal points, rather than physical lens surfaces.
Lens Setup

• In OSLO
  – Solve
    • Axial ray height solve (PY solve)
      – To specify the last thickness (the one before the image surface)
    • Chief ray height solve (PYC solve)
      – Final surface becomes the paraxial exit pupil
    • Angle solve
      – To constrain the f-number
  – Pickup
    • Force some surfaces to follow a fixing setting according to some specific surface (lens)