Chap31 Electromagnetic Induction

Electromagnetic induction (電磁感應) is the foundation of modern civilization.

Outline
- Historical note
- Evidence for E&M induction (induced emf)
- Lenz's law for induced current
- Faraday's Law – a law of nature
- Three ways to generate E-field
  - by electric charges
  - E-field due to charges separated by B-field (motional emf)
  - E-field created by changing B-field

Historical root
- 1820 Oersted: current (from E-field) → B

• Is it possible to generate current/E-field via B-field?
  -- 1821, Michael Faraday made a note that he will try to “convert magnetism to electricity.”

Discovery of E&M induction
- 1830 Joseph Henry (1797-1878)
  - Too busy to publish until 1831

• 1831 Michael Faraday (1791-1867)
Cases of electromagnetic induction

The key is at the ability to picture the spatial distribution of the B-field.

Only changing of flux matters!

Magnetic flux (磁通量)

for a plane: \( \Phi_B = \vec{B} \cdot \vec{A} \)

for a curvy surface: \( \Phi_B = \int \vec{B} \cdot d\vec{A} \)
**Lenz’s Law**

The effect of the induced emf is such as to oppose the change in flux that produces it.

**Example 31.3/page 634**

**EXAMPLE 31.3:** A metal rod of length \( \ell \) slides at constant velocity \( v \) on conducting rails that terminate in a resistor \( R \). There is a uniform and constant magnetic field perpendicular to the plane of the rails, as shown in Fig. 31.13. Find: (a) the current in the resistor; (b) the power dissipated in the resistor; (c) the mechanical power needed to pull the rod.

\[
\Phi = \bar{B} \cdot \bar{A} = B \ell x \quad |\varepsilon| = \left| \frac{d\Phi}{dt} \right| = B \ell \frac{dx}{dt} = B \ell v
\]

\[
I = \frac{|\varepsilon|}{R} = B \ell v \frac{1}{R} ; P_{\text{ele}} = I^2 R = \left( B \ell v \right)^2 \frac{1}{R}
\]

The rod experiences a force \( \vec{F} = I \ell \times \vec{B} \) (directed opposite to \( V \))

\[
\vec{F}_{\text{ext}} = -\vec{F}_B = -I \ell \times \vec{B} \Rightarrow P_{\text{mech}} = |\vec{F}_{\text{EXT}}| \cdot V = \left( B \ell v \right)^2 \frac{1}{R}
\]

**Generators (發電機)**

Grand Coulee Dam

電力→機械能→電磁感應→電能
**Concept of a generator**

To run a DC device, the current needed to be rectified using a commutator. To reduce the fluctuation further, a multiple coils with a multielement commutator has to be used, as in (c).

**Origin of the Induced emf**

The induced electric field motional emf is defined as the work done per unit charge by a source of emf as the charge moves around a closed loop.

\[
\varepsilon = \frac{W_{\text{me}}}{q} = \frac{1}{q} \oint \vec{F} \cdot d\vec{l} = \frac{1}{q} \oint (\vec{E} + \nabla \times \vec{B}) \cdot d\vec{l}
\]

\[
\therefore \varepsilon = \oint \vec{E} \cdot d\vec{l} + \oint (\nabla \times \vec{B}) \cdot d\vec{l}
\]

**The induced E-field**

When there is no relative motion, only explicit time dependence of the magnetic field contribute to \( \frac{d\Phi}{dt} \)

\[
\varepsilon = \oint \vec{E} \cdot d\vec{l} = - \frac{d\Phi}{dt} = -A \frac{dB}{dt} \text{ (if } A = \text{ constant)}
\]

There is an induced electric field in any closed path through which the magnetic field is changing.

- The induced \( \vec{E} \) lines are closed loops.
- The induced \( \vec{E} \) is a nonconservative field (\( \oint \vec{E} \cdot d\vec{l} \neq 0 \))

**Example 31.6**

The current in an ideal solenoid of radius \( R \) varies as a function of time. Find the induced electric field inside and outside the solenoid. Express the results in terms of \( \frac{dB}{dt} \)

(i) \( r < R \)

\[
\oint \vec{E} \cdot d\vec{l} = 2\pi r E = -\frac{d}{dt} \Phi = -(\pi r^2) \frac{dB}{dt}
\]

\[
E = -\frac{r dB}{2 \frac{dt}{dt}}
\]

(ii) \( r > R \)

\[
2\pi r E = -(\pi R^2) \frac{dB}{dt}
\]

\[
E = -\frac{R^2 dB}{2r \frac{dt}{dt}}
\]
Motional emf

When the magnetic field is constant in time

\[ \varepsilon = \oint (\mathbf{V} \times \mathbf{B}) \cdot d\ell = -\frac{d\Phi}{dt} \]

\[ \mathbf{F}_E = -e\mathbf{E}_0 = -\mathbf{F}_B = -(e\mathbf{V} \times \mathbf{B}) \]

\[ \mathbf{E}_0 = VB \]

\[ V_b - V_a = E_0 \ell = B\ell V \]

\[ \varepsilon = B\ell V \] Motional emf

Eddy current (渦電流)

Lenz’s law → eddy currents

Induction heating – hot plate
Auxiliary braking system

The eddy currents produce effective magnetic poles on the plate, gives a repulsive force that opposes the motion of the pendulum

the plate swings more freely through the magnetic field

(increase R ⇒ decrease I ⇒ P = IV ↓)

Metal detection (金屬感測)

(a) A magnet suspended above a rapidly spinning metal disk induces eddy current in the disk.

(b) When a current is switched on, the ring is repelled and flies vertically upward.

Other applications/demo

(a) Induced current due to eddy currents
(b) Induced current due to the transmitter coil

Receiver coil

Transmitter coil

Eddy currents in the metal reduce the induced current in the receiver coil.

Auxiliary braking system of train

Iron bar

Rowland ring
In a uniform magnetic field the magnetic flux through a plane area $\mathbf{A}$ is given by

$$\Phi = \mathbf{B} \cdot \mathbf{A}$$

If the area is not flat or the field is not uniform, the flux is given by

$$\Phi = \int \mathbf{B} \cdot d\mathbf{A}$$

**Faraday's law** of electromagnetic induction relates the induced emf in a closed loop to the rate of change of flux through it:

$$\varepsilon = -\frac{d\Phi}{dt}$$

The negative sign takes into account the direction of $\varepsilon$, which is given by **Lenz's law**: The induced emf opposes the change in magnetic flux that produces it. In a circuit, the induced field produced by the induced current may either oppose or reinforce the external field.

In general, emf is defined as the work done by a nonelectrostatic agent in carrying a unit charge around a closed loop:

$$\varepsilon = \frac{W}{q} = \oint \mathbf{F} \cdot d\ell$$

The Lorentz force on a charge is given by $\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})$. Thus,

$$\varepsilon = \oint (\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot d\ell$$

The first term is associated with an *induced electric field* and arises when the magnetic field depends explicitly on time. The second term arises when a conductor moves relative to a magnetic field. This *motional emf* arises from the magnetic force on a moving charge.